Design: bar pendulum with thickness *t*, mass *M*, distance from rotation center to center of mass L_{CM}, and overall length *L* and servo motor of torque τ_{SM} with horn of length ℓ_{horn}.

Simple Pendulum Model:

$$\ddot{\theta} + \frac{g}{L}\sin(\theta) = 0 \rightarrow \ddot{\theta} = -\frac{g}{L}\sin(\theta)$$

Incorporating Generic Losses due to Frictional Moments in Bearings:

$$\ddot{\theta} = -\frac{g}{L}\sin(\theta) - k_f sign(\dot{\theta})$$

Accounting for Torque due to Drag:

$$\tau_D = \int_{r=0}^{r=L} \frac{1}{2} C_D \rho r v^2 dA = \frac{1}{2}\rho\dot{\theta}^2 t \int_0^L C_D r^3 dr$$

$$[Re] = \frac{\rho t v}{\mu}$$

$$\rightarrow [Re]_{max} = \frac{\rho_{air} t v_{max}}{\mu_{air}} = \frac{\rho_{air} t L_{CM} \dot{\theta}_{max}}{\mu_{air}} = \frac{\rho_{air} t L_{CM_{max}} \sqrt{\frac{2g}{L_{CM_{max}}}\cos(\theta_{min} - \theta_{max})}}{\mu_{air}}$$

$$\rightarrow [Re]_{max} = \frac{\rho_{air} t L_{CM} \sqrt{\frac{2g}{L_{CM}}}}{\mu_{air}} = \frac{1.225\frac{kg}{m^3} * 6mm * 150mm * \sqrt{\frac{2 * 9.8\frac{m}{s^2}}{150mm}}}{18 * 10^{-6}\, Pa * s} = 700$$

$$\therefore [Re]_{max} \ll 1000, flow\ is\ fully\ laminar \rightarrow C_D \propto \frac{1}{[Re]}$$

$$\therefore C_D = \frac{k_c \mu}{\rho t v} \mid k_c \approx 24\ (approx.\ starting\ value\ based\ on\ ellipsoid, to\ be\ calibrated)$$

$$\rightarrow \tau_D = \frac{k_d \mu_{air}}{2}\dot{\theta}\int_0^L r^2 dr = \frac{k_d \mu_{air}}{6}\dot{\theta} L^3$$

$$\therefore \ddot{\theta} = -\frac{g}{L}\sin(\theta) - k_f sign(\dot{\theta}) - \frac{k_d \mu_{air}}{6 M L_{CM}^2}\dot{\theta} L^3$$

Accounting for Torque Applied by Servo Motor (when applicable):

$$\therefore \ddot{\theta} = \frac{\tau_{SM} L}{M L_{CM}^2 \ell_{horn}} - \frac{g}{L}\sin(\theta) - k_f sign(\dot{\theta}) - \frac{k_d \mu_{air}}{6 M L_{CM}^2}\dot{\theta} L^3$$